constant; f and F, arbitrary functions; S_1 and S_2, functions linearly related to the concentrations; x, space coordinate; t, time; and ψ, distribution factor.

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RETRIEVAL OF THE BOUNDARY CONDITIONS FROM THE TEMPERATURE MEASUREMENTS AT POINTS INSIDE A SYSTEM OF PLANE DOUBLE-LAYER BODIES

A. A. Shmukin, S. V. Pavlyuk, and N. M. Lazuchenkov

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A reverse heat-conduction problem is reduced to integrating a system of ordinary differential equations by the method of smoothing splines.

In many specific practical engineering problems there arises the situation where determining the temperature and the thermal flux at external surfaces in systems of plane double-layer bodies requires measurement of the temperature as a function of time at internal points of the system [1]. We will consider the rather general formulation of reverse boundary-value problems of heat conduction for double-layer plates with an ideal thermal contact at the joint. For obtaining correct solutions to the reverse heat-conduction problems we will use the solution to the Cauchy problem [2, 3] and the method of smoothing splines [4].

The heat-conduction equation in a Cartesian system of coordinates, independent for each plate, will be written as [5]

$$\frac{\partial T_k}{\partial \tau} = \varepsilon_k \frac{\partial^2 T_k}{\partial X^2}, \quad 0 \leqslant \tau < \infty, \quad 0 \leqslant X \leqslant X_k, \tag{1}$$

where k = 1, 2 is the consecutive number of each layer, $X = x/R_0$, $\tau = a_0 t/R_0^2$, $\varepsilon_k = a_k/a_0$, $X_k = R_k/R_0$, a_0 and R_0 are arbitrary values of, respectively, the thermal diffusivity and the geometrical dimension, and R_k is the thickness of the k-th layer.

The conditions of ideal contact and the initial conditions will be stipulated as

$$T_1|_{X=X_{p,1}} = T_2|_{X=X_{p,2}}, \tag{2}$$

$$\pm \frac{\lambda_1}{R_0} \left.\frac{\partial T_1}{\partial X}\right|_{X=X_{p,1}} = \frac{\lambda_2}{R_0} \left.\frac{\partial T_2}{\partial X}\right|_{X=X_{p,2}}, \tag{3}$$

$$T_k|_{\tau=0} = \varphi_k(X), \tag{4}$$

where in expression (3) the plus sign corresponds to systems of coordinates in series and the minus sign corresponds to systems of coordinates in opposition, $\varphi_k(X)$ in expression (4) characterizing a nonuniform temperature distribution.

We assume that the heat transfer between the surface of the first plate layer and the ambient medium is subject to a boundary condition of the second kind

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$$-\frac{\lambda_1}{R_0}\left.\frac{\partial T_1}{\partial X}\right|_{X=0}=q_1(\tau), \tag{5}$$

and the condition at the section of the other plate layer at point $X = X_2^*$ $(0 < X_2^* \leqslant X_2)$ is

$$T_2|_{X=X_2^*}=f_{e,2}(\tau), \tag{6}$$

where $f_{e,2}(\tau)$ is a function of time known from experiments.

The problem is to determine from Eqs. (1)-(6) the temperature functions within region $D_k = \{(X, \tau): 0 \leqslant X \leqslant X_k,\ 0 < \tau \leqslant \tau_k\}$ (k = 1, 2), and the thermal flux at the external surface of the second plate layer. For constructing the solution to this problem we use the auxiliary solution to the noncharacteristic Cauchy problem [2, 3]. The Cauchy condition will be stipulated at points coinciding with the origins of coordinates (we consider systems of coordinates in series)

$$T_k|_{X=0}=f_k(\tau), \tag{7}$$

$$-\frac{\lambda_k}{R_0}\left.\frac{\partial T_k}{\partial X}\right|_{X=0}=q_k(\tau). \tag{8}$$

The solution to the Cauchy problem (1), (7), (8), written in the form [2, 3]

$$T_k(X,\ \tau)=\sum_{n=0}^{\infty}\frac{(X/\sqrt{\varepsilon_k})^{2n}}{(2n)!}f_k^{(n)}(\tau)-\frac{R_0\sqrt{\varepsilon_k}}{\lambda_k}\sum_{n=0}^{\infty}\frac{(X/\sqrt{\varepsilon_k})^{2n+1}}{(2n+1)!}q_k^{(n)}(\tau), \tag{9}$$

exists and is unique in the class of analytic functions [5].

In solution (9) functions $f_k(\tau)$ and $q_k(\tau)$ characterize the variation of temperature and thermal flux at the external surface of the first plate layer and at the contact surface between layers. Using conditions (2), (3) ($X_{p,k} = 0$), we eliminate functions $f_2(\tau)$, $q_2(\tau)$ from solution (9) by expressing them through functions $f_1(\tau)$, $q_1(\tau)$. With condition (5), we now obtain

$$T_k(X,\ \tau)=\sum_{n=0}^{\infty}\Omega_{k,n}(X)f_1^{(n)}(\tau)-\frac{R_0\sqrt{\varepsilon_k}}{\lambda_k}\chi_k(X,\ \tau), \tag{10}$$

where

$$\chi_k(X,\ \tau)=\sum_{n=0}^{\infty}\omega_{k,n}(X)q_1^{(n)}(\tau),$$

$$\Omega_{1,n}(X)=(X/\sqrt{\varepsilon_1})^{2n}/(2n)!,\quad \omega_{1,n}(X)=(X/\sqrt{\varepsilon_1})^{2n+1}/(2n+1)!,$$

$$\Omega_{2,n}(X)=\frac{1}{2}\left\{(1+\nu_{1,2})\frac{(X/\sqrt{\varepsilon_2}+X_1/\sqrt{\varepsilon_1})^{2n}}{(2n)!}+(1-\nu_{1,2})\frac{(X/\sqrt{\varepsilon_2}-X_1/\sqrt{\varepsilon_1})^{2n}}{(2n)!}\right\},$$

$$\omega_{2,n}(X)=\frac{1}{2}\left\{(1+\nu_{1,2})\frac{(X/\sqrt{\varepsilon_2}+X_1/\sqrt{\varepsilon_1})^{2n+1}}{(2n+1)!}+(1-\nu_{1,2})\frac{(X/\sqrt{\varepsilon_2}-X_1/\sqrt{\varepsilon_1})^{2n+1}}{(2n+1)!}\right\}.$$

Here $\chi_k(X,\ \tau)$ is a given function, $f_1(\tau)$ is an unknown function, and $\nu_{1,2} = (\lambda_1/\lambda_2\sqrt{a_2/a_1}$ is a dimensionless parameter. For determining function $f_1(\tau)$ we will use condition (6). Inserting expression (10) into condition (6) yields an ordinary differential equation for $f_1(\tau)$, viz.,

$$\lim_{N\to\infty}\sum_{n=0}^{N}\Omega_{2,n}(X_2^*)f_1^{(n)}(\tau)=f_{e,2}(\tau)+\frac{R_0\sqrt{\varepsilon_2}}{\lambda_2}\chi_2(\tau,\ X_2^*). \tag{11}$$

We now introduce new unknown functions

$$Y_1=\Omega_{2,0}(X_2^*)f_1(\tau),\ Y_2=\Omega_{2,1}(X_2^*)f_1'(\tau),\ \ldots,\ Y_N=\Omega_{2,N-1}(X_2^*)f_1^{(N-1)}(\tau).$$

Reduction of the infinite series (11) in n to n = N ($N \in Z$) results in the system of normal ordinary differential equations

$$dY_n/d\tau=\varepsilon_nY_{n+1}, \tag{12}$$

$$dY_N/d\tau = \varepsilon_N \left[f_9(\tau) + (R_0 \sqrt{\varepsilon_2}/\lambda_2)\, \chi_2(\tau,\ X_2^*) - \sum_{n=0}^{N-1} Y_{n+1} \right],$$

where $\varepsilon_n = \Omega_{2,n}/\Omega_{2,n-1}$. Considering that [3]

$$f_1^{(n)}(\tau)|_{\tau=0} = \varphi_1^{(2n)}(\tau)|_{X=0}, \quad n = 0,\ 1,\ 2,\ \ldots,$$

we find that

$$Y_{n+1}|_{\tau=0} = \Omega_{2,n}(X_2^*)\, \varphi_1^{(2n)}(X)|_{X=0}. \tag{13}$$

Consequently, relation (10) can be rewritten as

$$T_k(X,\ \tau) = \lim_{N\to\infty} \sum_{n=0}^{N-1} \overline{\Omega}_{k,n}(X)\, Y_{n+1} - (R_0 \sqrt{\varepsilon_k}/\lambda_k)\, \chi_k(X,\ \tau), \tag{14}$$

where $\overline{\Omega_{k,n}}(X) = \Omega_{k,n}(X)/\Omega_{2,n}(X_2^*)$.

Solution (14), with Y_{n+1} determined through integration of the system of ordinary differential equations (12), is the solution to problem (1)-(6) for region D_k. Parameter N in expression (14) assumes the values N = 1, 2, 3, ... so that a set of approximate values can be obtained which steadily converge to the exact solution at $N \to \infty$. This approach to constructing the solution to problem (1)-(6), unlike the analytical methods [6], avoids a solution of intricate multiparameter transcendental equations and is, moreover, more optimal with respect to computer time than numerical methods [3]. Solution of a reverse boundary-value problem of heat conduction reduces here to extrapolation of solution (14) to the region above point X_2^*. Accordingly, the temperature and the thermal flux at the external surface of the second plate layer are determined from solution (14):

$$\begin{aligned} f_2(\tau) &= \lim_{N\to\infty} \sum_{n=0}^{N-1} \overline{\Omega}_{2,n}(X_2)\, Y_{n+1} - (R_0 \sqrt{\varepsilon_2}/\lambda_2)\, \chi_2(X_2,\ \tau), \\ q_2(\tau) &= (\lambda_2/R_0) \left[\lim_{N\to\infty} \sum_{n=0}^{N-1} \overline{\Omega}'_{2,n}(X_2)\, Y_{n+1} - \sqrt{\varepsilon_2}\, \chi'_2(X_2,\ \tau) \right], \end{aligned} \tag{15}$$

where each Y_{n+1} satisfies system (12) of ordinary differential equations.

We will now consider another variant of solution of the problem, one which avoids extrapolation. In opposing systems of coordinates, the Cauchy conditions (7) and (8) characterize the variation of temperatures and thermal fluxes given at the external surfaces of the plate. It is easily ascertained that the solution to the Cauchy problem on this basis is solution (9). This solution, together with the conditions of coupling (2) and (3) (the latter condition taken with a minus sign), yields the values of functions $f_1(\tau)$ and $q_1(\tau)$

$$f_1(\tau) = \sum_{n=0}^{\infty} Z_n f_2^{(n)}(\tau) - (R_0 \sqrt{\varepsilon_2}/\lambda_2) \sum_{n=0}^{\infty} \overline{Z}_n q_2^{(n)}(\tau), \tag{16}$$

$$q_1(\tau) = (\lambda_1/R_0\sqrt{\varepsilon_1}) \left[\sum_{n=0}^{\infty} \beta_n f_2^{(n+1)}(\tau) - \frac{R_0\sqrt{\varepsilon_2}}{\lambda_2} \sum_{n=0}^{\infty} \overline{\beta}_n q_2^{(n)}(\tau) \right], \tag{17}$$

where

$$Z_n = \sum_{j=0}^{n} A_{1,j} A_{2,n-j} + \nu_{2,1} \sum_{j=0}^{n+1} B_{1,j} B_{2,n-1-j},$$

$$\overline{Z}_n = \sum_{j=0}^{n} A_{1,j} B_{2,n-j} + \nu_{2,1} \sum_{j=0}^{n} B_{1,j} A_{2,n-j};$$

$$\beta_n = \sum_{j=0}^{n} B_{1,j} A_{2,n-j} + \nu_{2,1} \sum_{j=0}^{n} A_{1,j} B_{2,n-j},$$

$$\overline{\beta}_n = \sum_{j=0}^{n-1} B_{1,j} B_{2,n-1-j} + \nu_{2,1} \sum_{j=0}^{n} A_{1,j} A_{2,n-j};$$

$$A_{1,n}=\frac{(X_1/\sqrt{\varepsilon_1})^{2n}}{(2n)!};\quad B_{1,n}=\frac{(X_1/\sqrt{\varepsilon_1})^{2n+1}}{(2n+1)!};$$

$$A_{2,n}=\frac{(X_2/\sqrt{\varepsilon_2})^{2n}}{(2n)!};\quad B_{2,n}=\frac{(X_2/\sqrt{\varepsilon_2})^{2n+1}}{(2n+1)!};$$

$$\nu_{1,2}=\frac{\lambda_1}{\lambda_2}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}};\quad \nu_{2,1}=\frac{\lambda_2}{\lambda_1}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}.$$

Inserting expressions (16) and (17) into solution (9) yields

$$T_k(X,\ \tau)=\sum_{n=0}^{\infty}\Omega_{k,n}(X)\,f_2^{(n)}(\tau)-(R_0\sqrt{\varepsilon_k}/\lambda_k)\sum_{n=0}^{\infty}\omega_{k,n}(X)\,q_2^{(n)}(\tau), \tag{18}$$

where

$$\Omega_{2,n}(X)=\frac{(X/\sqrt{\varepsilon_2})^{2n}}{(2n)!};\quad \omega_{2,n}(X)=\frac{(X/\sqrt{\varepsilon_2})^{2n+1}}{(2n+1)!};$$

$$\Omega_{1,n}(X)=\sum_{j=0}^{n}\frac{(X/\sqrt{\varepsilon_1})^{2j}}{(2j)!}Z_{n-j}-\sum_{j=0}^{n-1}\frac{(X/\sqrt{\varepsilon_1})^{2j+1}}{(2j+1)!}\beta_{n-1-j}; \tag{19}$$

$$\omega_{1,n}(X)=\nu_{1,2}\left[-\sum_{j=0}^{n}\frac{(X/\sqrt{\varepsilon_1})^{2j+1}}{(2j+1)!}\bar{\beta}_{n-j}+\nu_{1,2}\sum_{j=0}^{n}\frac{(X/\sqrt{\varepsilon_1})^{2j}}{(2j)!}\bar{Z}_{n-j}\right].$$

Now, with relation (18) and condition (5), (6) taken into account, we obtain for $f_2(\tau)$ and $q_2(\tau)$ the system of ordinary differential equations

$$\sum_{n=0}^{\infty}\Omega_{2,n}(X_2^*)\,f_2^{(n)}(\tau)-(R_0\sqrt{\varepsilon_2}/\lambda_2)\sum_{n=0}^{\infty}\omega_{2,n}(X_2^*)\,q_2^{(n)}(\tau)=f_{e,2}(\tau),$$

$$\sum_{n=0}^{\infty}\beta_n f_2^{(n+1)}(\tau)-(R_0\sqrt{\varepsilon_2}/\lambda_2)\sum_{n=0}^{\infty}\bar{\beta}_n q_2^{(n)}(\tau)=(R_0\sqrt{\varepsilon_1}/\lambda_1)\,q_1(\tau). \tag{20}$$

Let us split Eqs. (20) so as to separate the functions $f_2(\tau)$ and $q_2(\tau)$. Upon multiplying each by the corresponding differential operators and then subtracting the first one from the second one, we obtain

$$\lim_{N\to\infty}\sum_{n=0}^{N}D_n f_2^{(n)}(\tau)=u_1(\tau),\quad \lim_{N\to\infty}\sum_{n=0}^{N}D_n q_2^{(n)}(\tau)=u_2(\tau), \tag{21}$$

where

$$D_n=\sum_{j=0}^{n}\frac{(X_2^*/\sqrt{\varepsilon_2})^{2j}}{(2j)!}\bar{\beta}_{n-j}-\sum_{j=0}^{n-1}\frac{(X_2^*/\sqrt{\varepsilon_2})^{2j+1}}{(2j+1)!}\beta_{n-1-j};$$

$$u_1(\tau)=\sum_{n=0}^{\infty}\bar{\beta}_n f_e^{(n)}(\tau)-(R_0\sqrt{\varepsilon_1}/\lambda_1)\sum_{n=0}^{\infty}\frac{(X_2^*/\sqrt{\varepsilon_2})^{2n+1}}{(2n+1)!}q_1^{(n)}(\tau);$$

$$u_2(\tau)=(\lambda_2/R_0\sqrt{\varepsilon_2})\left\{\sum_{n=0}^{\infty}\beta_n f_e^{(n+1)}(\tau)-(\lambda_2/R_0\sqrt{\varepsilon_2})\sum_{n=0}^{\infty}\frac{(X_2^*/\sqrt{\varepsilon_2})^{2n}}{(2n)!}q_1^{(n)}(\tau)\right\}.$$

We now introduce new unknown functions

$$Y_1=D_0f_2,\quad Y_2=D_1f_2',\ \ldots,\ Y_N=D_{N-1}f_2^{(N-1)},$$
$$\bar{Y}_1=D_0q_2,\quad \bar{Y}_2=D_1q_2',\ \ldots,\ \bar{Y}_N=D_{N-1}q_2^{(N-1)}. \tag{22}$$

As a result, system of equations (21) is replaced with the system of normal ordinary differential equations

$$dY_n/d\tau = \varepsilon_n Y_{n+1},$$

$$dY_N/d\tau = \varepsilon_N \left[u_1(\tau) - \sum_{n=0}^{N-1} Y_{n+1}\right], \tag{23}$$

$$d\overline{Y}_n/d\tau = \varepsilon_n \overline{Y}_{n+1},$$

$$d\overline{Y}_N/d\tau = \varepsilon_N \left[u_2(\tau) - \sum_{n=0}^{N-1} \overline{Y}_{n+1}\right], \tag{24}$$

where $\varepsilon_n = D_n/D_{n+1}$.

The initial conditions for system (23), (24) will be stipulated as [3]

$$Y_{n+1}|_{\tau=0} = D_n \varphi_2^{(2n)}(X)|_{X=0},$$

$$\overline{Y}_{n+1}|_{\tau=0} = (\lambda_2/R_0) D_n \varphi_2^{(2n+1)}(X)|_{X=0}. \tag{25}$$

Finally, the solution to problem (1)-(6) becomes

$$T_k(X,\ \tau) = \lim_{N\to\infty} \left[\sum_{n=0}^{N-1} \overline{\Omega}_{k,n}(X) Y_{n+1} - (R_0\sqrt{\varepsilon_k}/\lambda_k) \sum_{n=0}^{N-1} \overline{\omega}_{k,n}(X) \overline{Y}_{n+1}\right], \tag{26}$$

where

$$\overline{\Omega}_{k,n}(X) = \Omega_{k,n}(X)/D_n; \quad \overline{\omega}_{k,n}(X) = \omega_{k,n}(X)/D_n.$$

Thus, the sought temperature of the external surface and the thermal flux supplied to it are found directly by integration of the independent systems of ordinary differential equations (23) and (24), respectively, with the initial conditions (25).

The just-described method of solving reverse boundary-value heat-conduction problems can be easily extended also to the problem of retrieving the boundary conditions from measurement of the temperature at two internal points in the system.

We replace the boundary condition (5) with a boundary condition of the first kind, viz.,

$$T_1|_{X=X_1^*} = f_{e,1}(\tau), \quad 0 < X_1^* < X_1, \tag{27}$$

where function $f_{e,1}(\tau)$ characterizes the variation of the temperature with time and is known from experiments. We will then use solution (9) to the Cauchy problem, transformed to expression (18). Inserting expression (18) into conditions (6) and (26) yields a system of ordinary differential equations for the sought unknown functions, viz.,

$$\sum_{n=0}^{\infty} \Omega_{n,k}(X_k^*) f_2^{(n)}(\tau) - (R_0\sqrt{\varepsilon_k}/\lambda_k) \sum_{n=0}^{\infty} \omega_{k,n}(X_k^*) q_2^{(n)}(\tau) = f_{e,k}(\tau), \tag{28}$$

where k = 1, 2 and all other symbols are the same as in expression (18).

Splitting Eq. (28) so as to separate $f_2(\tau)$ and $q_2(\tau)$, we obtain a relation of the (21) kind with D_n sequences and $u_k(\tau)$ functions

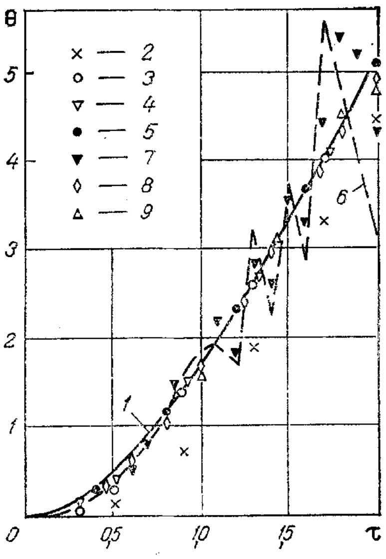
$$D_n = \nu_{2,1} \sum_{j=0}^{n} \omega_{1,j}(X_1^*) \Omega_{2,n-j}(X_2^*) - \sum_{j=0}^{n} \omega_{2,j}(X_2^*) \Omega_{1,n-j}(X_1^*),$$

$$u_1(\tau) = \nu_{2,1} \sum_{n=0}^{\infty} \omega_{1,n}(X_1^*) f_{e,2}^{(n)}(\tau) - \sum_{n=0}^{\infty} \omega_{2,n}(X_2^*) f_{e,1}^{(n)}(\tau), \tag{29}$$

$$u_2(\tau) = (\lambda_2/R_0\sqrt{\varepsilon_2}) \left[\sum_{n=0}^{\infty} \Omega_{1,n}(X_1^*) f_{e,2}^{(n)}(\tau) - \sum_{n=0}^{\infty} \Omega_{2,n}(X_2^*) f_{e,1}^{(n)}(\tau)\right].$$

The solution to the problem is now finally transformed to expression (26), where each Y_{n+1} and $\overline{Y}_{n+1}$ is determined through integration of system (23), (24) of ordinary differential equations with the initial conditions (25) and relations (29).

In connection with a stability analysis of these solutions to the reverse problem of heat conduction, it must be noted that incorrectness manifests itself in the obtained solu-



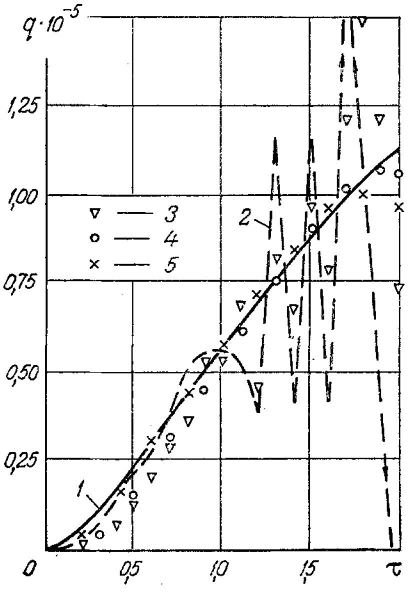


Fig. 1 Fig. 2

Fig. 1. Retrieval of the temperature of the heating surface from input data without perturbation and with low-intensity perturbation: 1) exact solution [7]; 2-4) for N = 2, 3, 4, respectively, with extrapolation and without perturbation; 5) for N = 2, 3, 5 without extrapolation and perturbation; 6, 7) for N = 3 and ε = 5%, unregularized solution, respectively, without and with extrapolation; 8, 9) for N = 3 and ε = 5% with regularization and, respectively, without and with extrapolation.

Fig. 2. Retrieval of the thermal flux at the heating surface from input temperature data with low-intensity perturbation: 1) exact solution [7]; 2, 3) for N = 3 and ε = 5% unregularized solution, respectively, without and with extrapolation; 4, 5) for N = 3 and ε = 5% with regularization and, respectively, without and with extrapolation.

TABLE 1. Thermophysical Characteristics of the Materials of Layers

Layer No.	R_k, m	a_k, m²/sec	λ_k, W/(m·deg)
1	$3\cdot10^{-3}$	$0,30864\cdot10^{-4}$	93,04
2	$5\cdot10^{-3}$	$0,69450\cdot10^{-4}$	116,3
0	$5\cdot10^{-3}$	$0,69450\cdot10^{-4}$	—

tions in various forms. Solution (15), e.g., is sensitive to inaccuracies in the input function $f_{e,2}(\tau)$. With a certain level of inaccuracy, there appears in the solution an oscillatory component of appreciable amplitude. Solution (15) depends also on the intensity and the dynamicity of the thermal experiment, as well as on how close to the surface the temperature probes have been placed. In solution (23)-(26) incorrectness manifests itself in the retrieval of derivatives of the grid function, this function being known from experiments. In order to arrive at correct solutions to reverse boundary-value heat-conduction problems, therefore, it is necessary to include in the algorithms constructed here some additional regularizing algorithm. An effective algorithm for this is that of smoothing with splines, which has been used already [4] for smoothing and thus obtaining a regularized solution to a differentiation problem.

Our algorithm of solving a reverse boundary-value heat-conduction problem, together with the algorithm of smoothing splines, has been written in ALGOL-60 language for a model BÉSM-4M high-speed computer. Methodical calculations were made for retrieval of the boundary

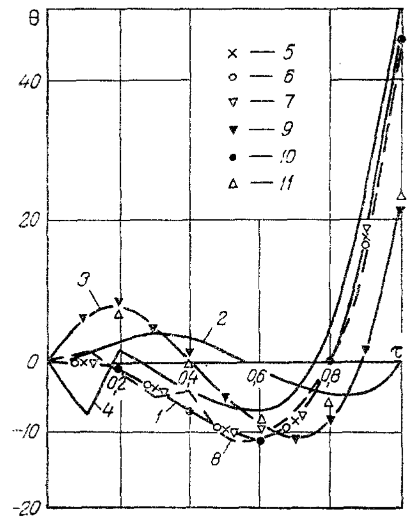


Fig. 3

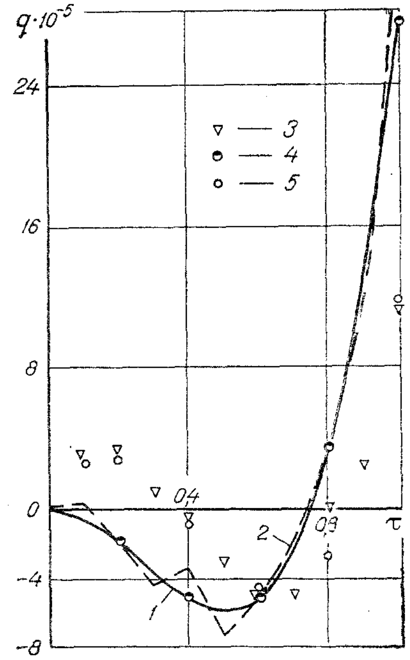


Fig. 4

Fig. 3. Retrieval of the temperature of the heating surface from thermal stress input data without and with perturbation: 1) exact solution [7]; 2-4) for N = 2, 3, 5, respectively, with extrapolation and without perturbation; 5-7) for N = 2, 3, 5, respectively, without extrapolation and perturbation; 8, 9) for N = 3 and ε = 5%, unregularized solution, respectively, without and with extrapolation; 10, 11) for N = 3 and ε = 5%, with regularization and, respectively, without and with extrapolation.

Fig. 4. Retrieval of the thermal flux at the heating surface from input temperature data with perturbation, in a process with thermal stress: 1) exact solution [7]; 2, 3) for N = 3 and ε = 5%, unregularized solution, respectively, without and with extrapolation; 4, 5) for N = 3 and ε = 5% with regularization and, respectively, without and with extrapolation.

conditions at the heating surface of a plane double-layer body. The external surface of the second layer was regarded as the heating surface and its internal surface on the side of the first layer as a thermally insulated one. The problem was solved in terms of relative temperatures $\Theta(X, \tau) = T(X,\tau)/T_0$ (T_0 = const) for various values of parameter N characterizing the accuracy of the mathematical model. At the first instant of time the system was assumed to be heating uniformly (with $\varphi_k(X) = 0$ in expression (4)), and the experimental temperature reading in solution (14), (15), (23)-(26) was stipulated at the contact between layers ($X_2^* = 0$, $X_2^* = X_2$). The thermophysical characteristics of the materials are given in Table 1.

The systems (12), (23), and (24) of ordinary differential equations were integrated by the Runge—Kutta method, with the integration step $\Delta\tau = 0.001$ throughout. For the purpose of testing the mathematical model, calculations were made with various exact values of the input temperature. The results of the solution are shown in Figs. 1 and 3 (curves 2-5 in Fig. 1 and curves 2-7 in Fig. 3). The results of solving the reverse problem of heat conduction with extrapolation for slowly evolving processes hardly differ from the results of its solution without extrapolation (Fig. 1). For intensive processes there appears a singularity on the first time interval, however, viz., series (14) does not converge here. On the same diagrams, also in Figs. 2 and 4, are shown the results of solving the reverse heat-transfer problem (with N = 3) for various intensities of perturbation of the input temperature and a uniform distribution of errors, solutions obtained with and without regularization. Perturbations of the input function were entered according to the relation

$$f_\delta(\tau) = f_e(\tau)(1 + \delta\varepsilon),$$

with $f_e(\tau)$ denoting the exact temperature, ε denoting the magnitude of the entered relative error, and δ denoting a random number generated by the random numbers generator to simulate the fluctuation errors of measurements. An analysis of these results confirms the effectiveness of the regularization algorithm and the reliability of the solution. A solution with extrapolation can, however, be recommended only for slowly evolving thermal processes. Increasing the relative error of input temperatures does not give rise to any other singularities in the solution to a reverse heat-conduction problem. As the analysis of results (Figs. 1 and 4) indicates, however, an inaccurate stipulation of the boundary condition for the regularizing spline gives rise to appreciable errors in the retrieved boundary functions within a finite time interval. In our case the condition $S''_{\Delta}(b) = 0$ was stipulated for the spline, this being correct for steady-state thermal conditions. At the end of the interval, therefore, this condition should correspond to steady-state thermal conditions.

NOTATION

T, temperature; t, time coordinate; x, space coordinate; and a, thermal diffusivity.

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ESTIMATES OF THE VALIDITY RANGE FOR THE HYPERBOLIC EQUATION OF HEAT-CONDUCTION IN HOMOGENEOUS SYMMETRIC CONTINUOUS BODIES

K. V. Lakusta and M. P. Lenyuk

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Estimates are made of the geometrical dimensions of symmetric continuous bodies, the temperature fields within which can be described by, respectively, the hyperbolic or the parabolic heat-conduction equation.

The phenomenological heat-conduction theory has been developed in a formulation uniform with respect to geometrical variables [1]. An analysis of heat and mass transfer, especially when the process is nonsteady and very intensive, leads to a hyperbolic heat-conduction equation and has served as a basis for a dynamic heat-conduction theory [2].

The problem of determining the structure of the temperature field in homogeneous symmetric continuous bodies reduces, within the dynamic theory of heat conduction, to that of finding within the region $D = [0, t_1] \times \Omega = \{(t, r),\ 0 \leqslant t \leqslant t_1,\ 0 \leqslant r \leqslant R\}$ a bounded and sufficiently smooth solution to the equation [2]

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